SUBTRACTION FACTS THAT STICK

Help Your Child Master the Subtraction Facts for Good in Just Eight Weeks

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WELL-TRAINED MIND PRESS

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PREFACE

My first job out of college was teaching fifth grade in an excellent public school in the Boston suburbs. I was thrilled to have the chance to work in such a great school district, and I could hardly wait to meet my students as I prepared for my first year of teaching.

Most of all, I was excited to teach math. I majored in math in college, and I eagerly looked forward to sharing my love of the subject with my students. Even before school began, I started to plan lessons covering the usual fifth-grade topics, like geometry, fractions, decimals, and percentages.

However, it only took a couple weeks into the school year for me to realize that some of my students needed a better mastery of the basics before they'd be ready to tackle fifth-grade work. My colleagues in the lower grades assured me that they'd taught the math facts diligently and encouraged parents to work on the facts at home. So why did I have bright ten-year-olds in my class who couldn't subtract 9 from 13?

As I probed further, I discovered that nearly all of my students had once memorized their math facts. But the facts just hadn't stuck. Their teachers and parents had conscientiously made flash cards and drilled the facts, over and over. This method had worked for some of the children. But for others, it seemed that these essential facts had gone straight into short-term memory and then straight out again.

So, instead of repeating a method that hadn't worked, I decided to try a different approach with my students. Instead of using rote memorization to master the facts, I taught my students how to visualize the numbers and use mental strategies to find the solutions. This took a little teaching time at the beginning, but the results were worth it. At first, it took my students a few seconds to apply the strategies. But with a little practice, the strategies became so automatic that my students "just knew" the answers—and became much more confident and successful in their math studies as a result.

That was nearly 15 years ago. Since then, I've taught several years of fifth-grade math classes, written math curricula, tutored students who struggle in math, and begun homeschooling my own children. Through these experiences, I've refined the approach I used with my first class of fifth-graders to create a simple, effective program that will help any child master the subtraction facts—all without flash cards or rote

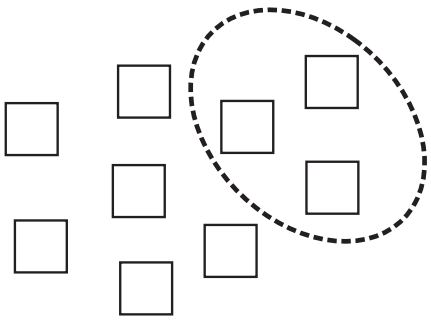
memorization. (Note that it's crucial that your child master the addition facts before tackling subtraction, since many subtraction strategies rely on being able to use "backwards addition." If your child has not yet learned the addition facts, I suggest you have your child work through *Addition Facts That Stick* first and then return to this book.)

Over the years, I've met so many parents who want to help their children master these important math foundations but just aren't sure how to do so effectively. That's why I've written this book. It will guide you step by step as you help your child master the subtraction facts, once and for all, so that they truly stick.

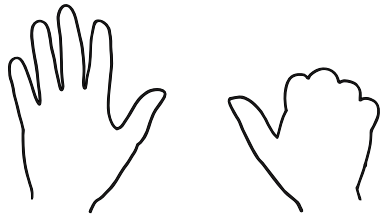
INTRODUCTION

Counting: The biggest obstacle to learning the subtraction facts

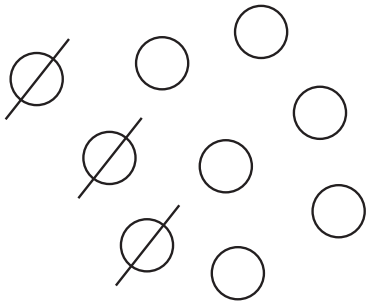
Remember when your child first started counting? Like me, you probably beamed with pride as your toddler painstakingly counted the crackers on her plate or the toy trains lined up on the carpet. Counting lays the foundation for understanding numbers, and it's an essential skill for little ones to learn. Then, as children grow, they use their counting skills to help them make sense of more complex math concepts, like subtraction. Counting and taking away objects, counting backwards on fingers, and counting crossed-out pictures all help children begin to understand what subtraction means.



"I had 9 blocks and
took away 3 of them."



"I held up 9 fingers and counted backwards 3."



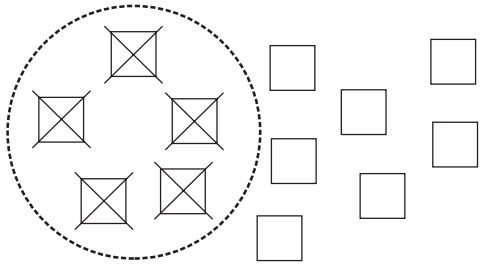
"I drew 9 circles and crossed out 3."

Three different ways children might use counting to understand 9 – 3.

But although counting helps children *understand the concept* of subtraction, counting can actually hold them back and prevent them from *mastering the subtraction facts*. Whether your child is a first-grader tackling the facts for the first time or a fourth-grader who never learned them well, she'll find it hard to become fluent with the subtraction facts if she relies too much on counting.

Why is counting such a problem?

To understand why counting creates such an obstacle, let's take a closer look at the thought process children use when they count to solve subtraction problems. As an example, imagine a child using blocks to find 12 – 5: First, he counts out 12 blocks. Then, he counts five of the blocks and takes them away. Finally, he counts the remaining seven blocks to find the answer.



Using blocks to find 12 – 5.

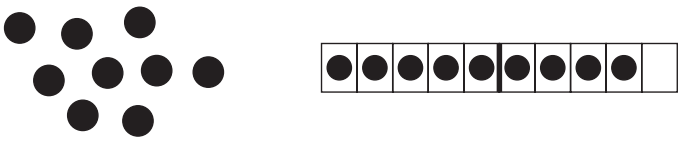
Obviously, this process is slow and inefficient. (Not to mention error-prone: it's easy to make a mistake when you're counting so many blocks!) But even more significantly, this child spends all his mental energy counting and keeping track of blocks. He doesn't have any brain space left over to pay attention to the relationships between the numbers. All he's doing is following a routine procedure, not building a mental framework for knowing the answers automatically.

Instead, children need a way to visualize numbers as *groups*, not collections of single objects that have to be counted. Visualizing numbers as groups allows a child to quickly recall key relationships between numbers (for example, that seven is two more than five, and three less than ten). Then, the child can use those relationships to find answers quickly, without counting. Fortunately, there's a simple tool that allows young learners to visualize numbers as groups: the ten-frame.

The solution: Visualizing numbers on the ten-frame

A ten-frame is a simple grid of ten squares, with a dark line dividing it in half. It may not look like much, but it's a powerful and versatile math tool.

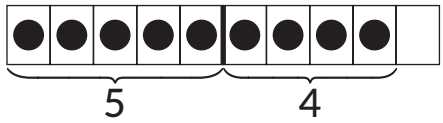
To understand why the ten-frame is so useful, compare these two sets of counters:



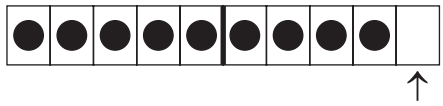
You can probably tell right away that there are nine counters on the ten-frame. But can you tell how many counters are in the scattered set without counting one by one?

(Probably not! There are nine in that set as well.) The structure of the ten-frame makes it much easier to recognize and visualize quantities.

The other benefit of the ten-frame is that it reveals important number relationships. The dividing line allows us to immediately see that 9 = 5 + 4, since there are five counters on one side of the line and four counters on the other side. We can also tell straightaway that nine is one less than ten, since there is only one empty box in the ten-frame.

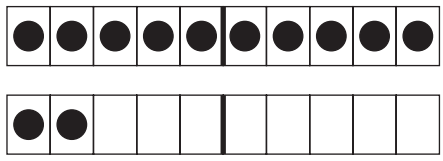


The ten-frame shows that 9 = 5 + 4.



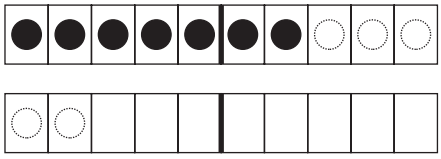
Since only one box is empty, nine must be one less than ten.

Once children learn to visualize numbers as groups on a ten-frame, they can then use their understanding of number relationships to solve subtraction problems efficiently and accurately. For example, to subtract 12 – 5, your child won't need to count out piles of counters or count backwards on her fingers. Instead, she will learn to imagine 12 counters organized on ten-frames.



Twelve counters on ten-frames. There are ten on the top ten-frame and two on the bottom ten-frame.

Now, she can simply imagine subtracting five in two steps: First, she visualizes removing two counters from the bottom row. Then, she visualizes removing three counters from the top row (for a total of five counters taken away). Now, it's easy to see that there are seven counters left, so 12 – 5 = 7.



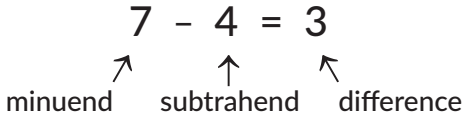
The ten-frame helps children visualize that 12 – 5 = 7.

You'll start by using physical counters on a ten-frame to teach your child how to figure out the answers. As your child becomes more comfortable with the ten-frame, you'll encourage her to begin to imagine it in her head to see the answers without using any actual physical counters. With practice, she'll see the answers so quickly that she'll "just know" the answers and will have the subtraction facts fully mastered.

How to use this book

Instead of overwhelming your child with all 81 subtraction facts, this program breaks the subtraction facts into eight units. Each unit targets a small group of subtraction facts that can be solved with the same mental strategy, and each unit is designed to take about a week. By the end of eight weeks, your child will have learned all the subtraction facts.

In subtraction, the number you're subtracting from (the first number in the problem) is called the minuend. The number you're subtracting (the second number in the problem) is called the subtrahend. The answer to a subtraction problem is called the difference. Your child doesn't need to know these terms, but I will use them sometimes in the instructions for simplicity's sake.



You'll use direct teaching, games, and written practice to help your child master the focus facts for each week. Here's how each week will look:

Day 1: Introduce new facts and teach a new game

On the first day of each week, you will use counters and a ten-frame to help your child learn to visualize numbers. You'll explain a new mental strategy to your child and help

your child practice applying the strategy to the week's focus facts. (Don't worry if you have never taught math before—this book will guide you step by step.)

Next, you will teach your child a fun game to practice with the focus facts. These fun games provide a lot of practice in a short amount of time. Even more importantly, they allow you to quickly correct any mistakes and monitor how well your child is using the new mental strategy.

Days 2-5: Play game and complete practice pages

For the rest of the week, you will play the new game again each day. As you play, you'll encourage your child to continue using the mental strategy introduced on Day 1.

Your child will also complete a short practice page each day. This will give your child practice at solving the week's subtraction facts in written form. The practice pages also review all the subtraction facts that your child has learned in previous weeks.

Teaching tips

- Schedule a consistent time each day for subtraction fact practice. You'll be less likely to forget, and your child will be less likely to argue. Try to choose a time when your child is alert and easily able to concentrate.
- Plan to work on the activities in this book for about 15 minutes each session, with five sessions per week. However, different children need different amounts of time to master each group of facts. Feel free to take as long as your child needs to master each unit.
- Discourage your child from counting to solve problems. As discussed above, counting prevents children from understanding the number relationships that lead to subtraction fact mastery. The only exception to this is Week 1, where your child will count back just one or two to solve the –1 and –2 facts.
- Keep the practice sessions positive, upbeat, and fast-paced. Have fun playing the games with your child, and enjoy the one-on-one time together.
- If your child is a reluctant writer, don't let writing difficulties interfere with mastering the subtraction facts. It's fine to have your child answer the worksheet problems orally rather than writing them.
- Many young children freeze when they feel time pressure. Unless your child is age ten or older, don't time him as he does the practice pages. For an older child, aim for your child to know each subtraction fact in three seconds or less.

Is your child ready to master the subtraction facts?

This book is designed for children who understand the concept of subtraction but do not yet know the answers to the subtraction facts automatically. While it's fine to introduce your younger child to the games and strategies, don't expect thorough mastery of the subtraction facts until your child is *at least* seven years old. Subtraction is substantially more difficult for most children than addition, and many children's brains aren't developmentally mature enough to master the subtraction facts until that age.

To be successful at mastering the subtraction facts, your child should first:

- Understand that subtraction can mean taking away or finding a difference. For example, 13 – 7 can mean, "How many are left when you take seven away from 13?" Or, 13 – 7 can be interpreted as, "How much more is 13 than seven?"
- Understand that subtraction is the opposite of addition.
- Know the addition facts up to 9 + 9. Many subtraction strategies rely on being able to use "backwards addition," so this is crucial. (If your child has not yet mastered the addition facts, work through *Addition Facts That Stick* first and then return to this book.)

If your child has these foundational skills in place, she is ready to master the subtraction facts.

What you'll need

All of the game boards and practice pages you'll need for this program are included in the back of the book. You'll also need a few everyday items to complete the activities and play the games:

- 20 small counters of two different colors (tiles, blocks, plastic bears, coins, etc.)
- Coin (any kind with heads and tails)
- Two game tokens
- Deck of regular playing cards
- Regular, six-sided die
- Paper and pencil

WEEK 2

SUBTRACTING THREE AND FOUR

WEEK 2 AT A GLANCE

Last week, you taught your child to subtract one or two by removing one or two counters from the ten-frame and counting backwards to find how many counters were left. This week, your child will learn to subtract three and four. She will continue to take away counters from the ten-frame, but she won't count backwards to find the remaining number of counters. Instead, she will use the structure of the ten-frame (especially the dark dividing line in the middle) to figure out how many counters are left.

Week 2 Focus Facts:

10 – 3 = 7
10 – 4 = 6
9 – 4 = 5
9 – 3 = 6
8 – 4 = 4
8 – 3 = 5
7 – 4 = 3
7 – 3 = 4
6 – 4 = 2
6 – 3 = 3
5 – 4 = 1
5 – 3 = 2
4 – 3 = 1

You will need:

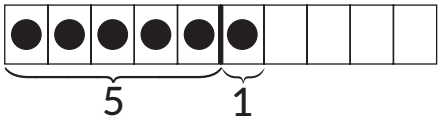
- Ten-frame cards 1-9 (page 71)
- Ten-frames (page 69)
- Paper and pencil
- 20 small counters of two different colors
- *Tic-Tac-Toe* game board (page 73)
- 6s, 7s, 8s, 9s, and 10s from a deck of cards (four of each, 20 total cards)
- Week 2 Practice Pages (pages 111–117)

DAY 1: NEW TEACHING

Warm-up activity

Show your child the ten-frame card with six circles.

"How many circles are there?" *Six.* Point out that the dark line in the middle divides the ten-frame into groups of five. "So, there are five circles on the left side of the frame, plus one more circle on the right side, for a total of six circles."



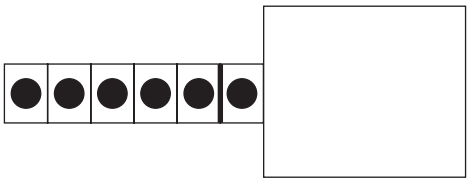
Discuss a couple more cards in this way to make sure your child understands what the cards show.

Shuffle all of the ten-frame cards. Flash each card for a second or two and ask how many circles there are. Adjust your pace to your child, and stop and allow more time to look at a card if needed. Encourage your child to use reasoning—not counting one by one—to figure out how many circles are on each card.

Introduce new facts

Write **10 − 4 =** on a piece of paper and place ten counters on the ten-frame.

"Imagine if I took away four of the counters." Cover the four counters farthest to the right with a blank piece of paper or your hand.

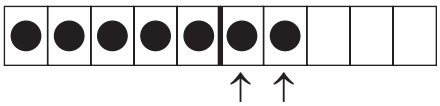


"How many counters would be left?" *Six.* Encourage your child to use reasoning—not counting one by one—to figure out how many counters are left on the ten-frame. Since there are five counters to the left of the dark line and one counter to the right of the dark line, there must be 5 + 1, or six counters.

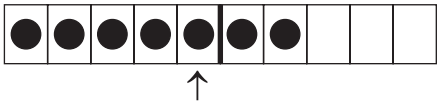
Have your child complete the written problem: **10 − 4 = 6.**

Write **7 − 3 =** on a piece of paper and place seven counters on the ten-frame.

"Imagine if I took away three of the counters. First, I'd take away the two counters on the right side of the dark line." (Point to these two counters.)



"Then, I'd still need to take away one more counter on the other side of the dark line." (Point to this counter.)



"How many counters would be left then?" *Four.* Have her remove the three counters to confirm her answer.

Have your child complete the written problem: **7 − 3 = 4.**

Have your child use counters to model the other subtraction facts for this week and find their answers. (They are listed on page 24.) Make sure she always fills in the ten-frame from left to right without skipping any boxes, and remind her to use reasoning—not counting—to find the number of counters that are left.

Play *Subtraction Tic-Tac-Toe*

Teach your child how to play *Subtraction Tic-Tac-Toe* and play several times.

MATERIALS

- *Subtraction Tic-Tac-Toe* game board (page 73)

7	4	3
6	5	6
4	3	5

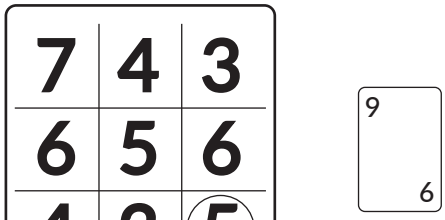
- 6s, 7s, 8s, 9s, and 10s from a regular deck of cards (four each; 20 cards total)
- Five small counters per player, with a different color for each player

OBJECT OF THE GAME

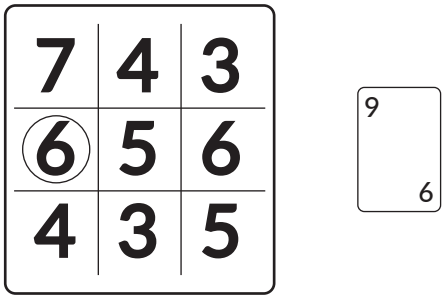
Be the first player to fill three boxes in a row (as in regular *Tic-Tac-Toe*).

HOW TO PLAY

Shuffle the cards and place the stack face down on the table. On your turn, flip over the top card. Subtract either three or four from the card and place one of your counters on the box that matches the difference. For example, if you draw a nine, you can subtract three from nine and place a marker on a box with a six. Or, you can subtract four from nine and place a marker on a box with a five.



One possible play if you draw a nine. Since 9 – 4 = 5, you can cover a five.



Another possible play if you draw a nine. Since 9 – 3 = 6, you can cover a six.

Play then passes to the other player. Continue until one player has completed an entire row, column, or diagonal.

Days 2-5: *Subtraction Tic-Tac-Toe* and Practice Pages

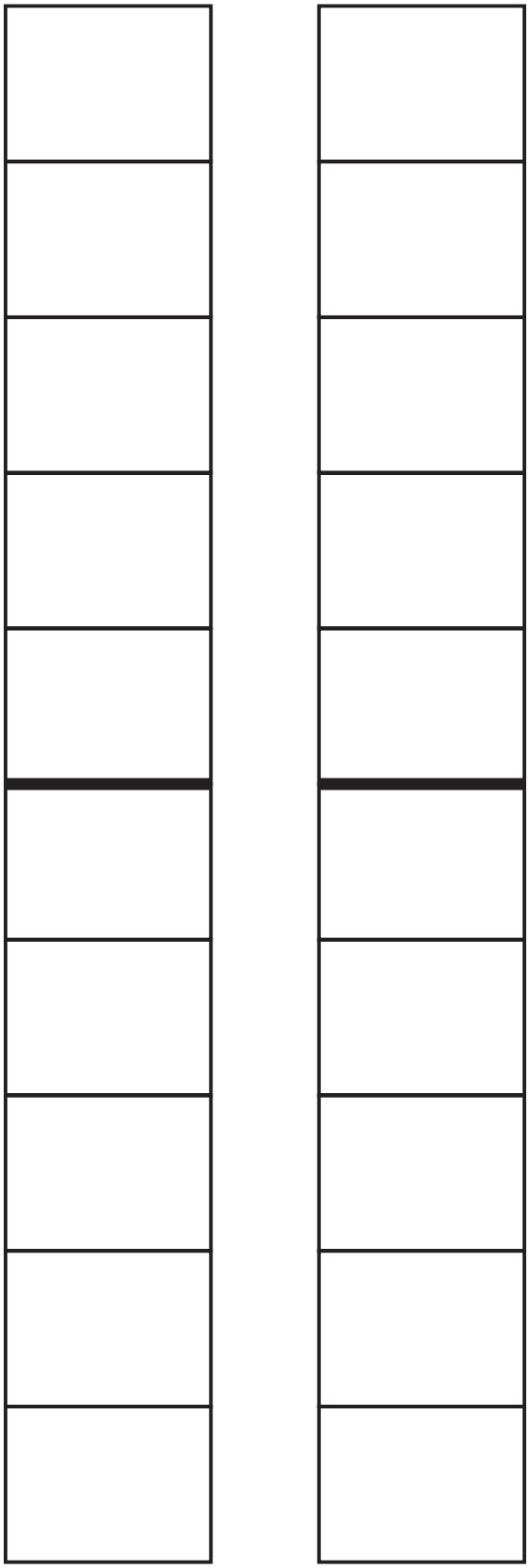
Each day, play *Subtraction Tic-Tac-Toe* several times. Then, have your child complete one Week 2 Practice Page to practice writing the answers to this week's subtraction facts and to review the facts learned so far. If needed, have her either put counters on the ten-frame or visualize counters to find the answers.

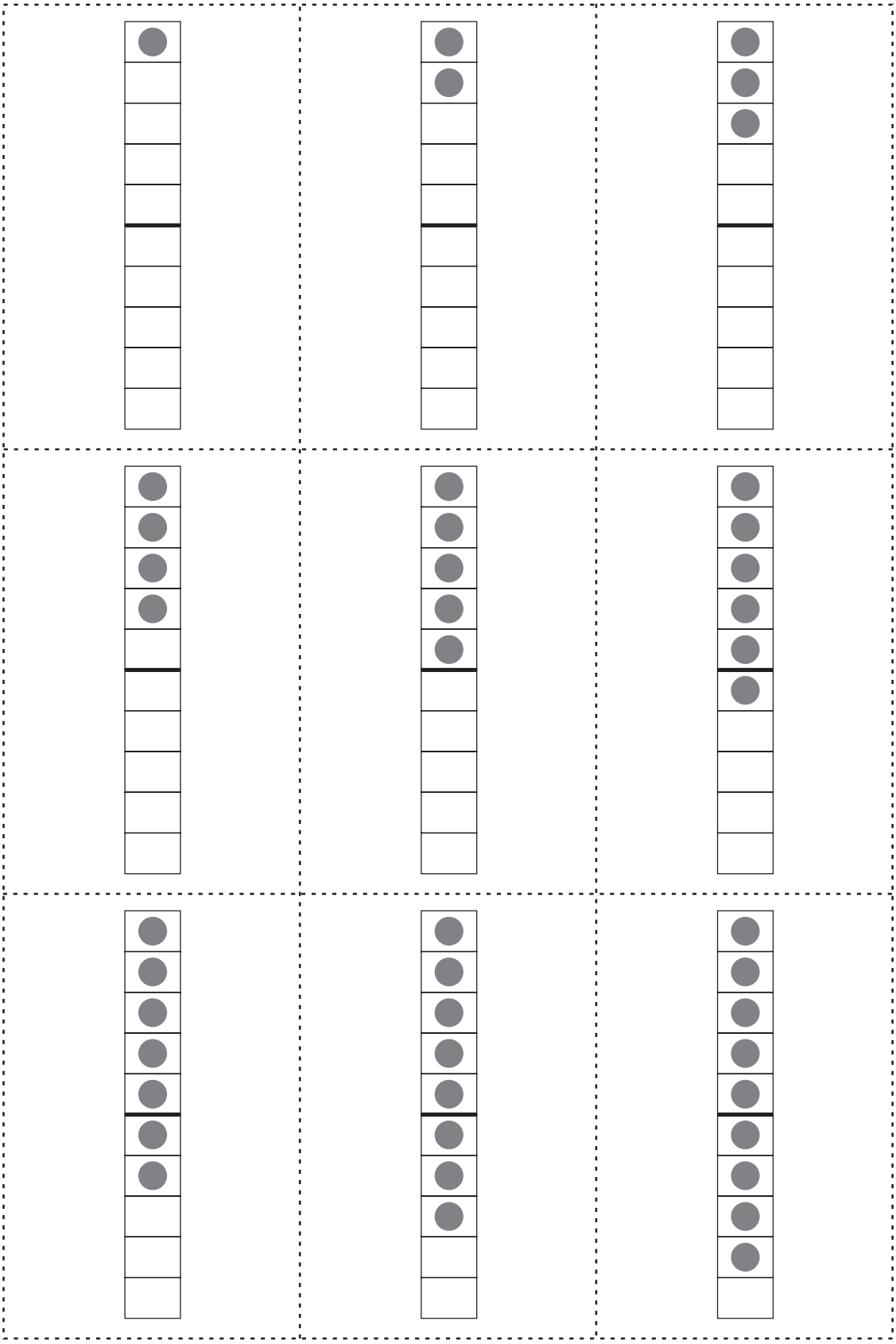
Over the course of the week, encourage your child to gradually transition from actual counters on the ten-frame to imagined counters in her head so that she learns to "see" the answers. Different children will progress at different speeds, but here is a sample of what this might look like:

- Day 2: Child acts out each step of each problem with physical counters.
- Day 3: Child represents the minuends (the first numbers in the problems) with physical counters and then imagines removing the subtrahends (the second numbers in the problems).
- Day 4: Child closes her eyes and visualizes the counters for most problems. She only occasionally uses physical counters for the more difficult problems.
- Day 5: Child solves nearly all the problems without counters.

If your child has trouble visualizing the counters, continue practicing this week's warm-up activity each day. Also have her practice looking at each card, closing her eyes, and then picturing the same arrangement. Remind her to pay close attention to the dark line in the middle of the ten-frame to help her visualize the numbers as groups.

Answers to this week's worksheets are on page 169.





Ten-Frames 1-9

Tic-Tac-Toe

7	4	3
6	5	6
4	3	5

Week 2

Practice Page 1

8 – 3 = _____	3 – 2 = _____	9 – 4 = _____
2 – 1 = _____	7 – 3 = _____	9 – 1 = _____
5 – 2 = _____	10 – 3 = _____	8 – 2 = _____
8 – 4 = _____	8 – 2 = _____	3 – 1 = _____
7 – 2 = _____	7 – 4 = _____	9 – 3 = _____
10 – 4 = _____	6 – 1 = _____	6 – 3 = _____

Week 2

Practice Page 2

8 – 4 = _____	9 – 2 = _____	8 – 1 = _____
10 – 2 = _____	7 – 4 = _____	9 – 3 = _____
10 – 4 = _____	5 – 1 = _____	6 – 3 = _____
8 – 3 = _____	6 – 2 = _____	9 – 4 = _____
4 – 2 = _____	7 – 3 = _____	10 – 1 = _____
4 – 1 = _____	10 – 3 = _____	7 – 1 = _____

Week 2

Practice Page 3

10 – 1 = _____	8 – 2 = _____	8 – 4 = _____
7 – 4 = _____	9 – 3 = _____	3 – 1 = _____
4 – 2 = _____	6 – 3 = _____	10 – 4 = _____
6 – 1 = _____	9 – 4 = _____	8 – 3 = _____
7 – 3 = _____	7 – 2 = _____	5 – 2 = _____
10 – 3 = _____	7 – 1 = _____	9 – 2 = _____

Week 2

Practice Page 4

10 – 2 = _____	6 – 3 = _____	10 – 4 = _____
5 – 1 = _____	9 – 4 = _____	8 – 3 = _____
9 – 1 = _____	6 – 2 = _____	8 – 4 = _____
7 – 4 = _____	9 – 3 = _____	7 – 2 = _____
7 – 3 = _____	2 – 1 = _____	4 – 1 = _____
10 – 3 = _____	8 – 1 = _____	3 – 2 = _____

Week 2		Practice Page 1
8 − 3 = 5	3 − 2 = 1	9 − 4 = 5
2 − 1 = 1	7 − 3 = 4	9 − 1 = 8
5 − 2 = 3	10 − 3 = 7	8 − 2 = 6
8 − 4 = 4	8 − 2 = 6	3 − 1 = 2
7 − 2 = 5	7 − 4 = 3	9 − 3 = 6
10 − 4 = 6	6 − 1 = 5	6 − 3 = 3

Week 2		Practice Page 2
8 − 4 = 4	9 − 2 = 7	8 − 1 = 7
10 − 2 = 8	7 − 4 = 3	9 − 3 = 6
10 − 4 = 6	5 − 1 = 4	6 − 3 = 3
8 − 3 = 5	6 − 2 = 4	9 − 4 = 5
4 − 2 = 2	7 − 3 = 4	10 − 1 = 9
4 − 1 = 3	10 − 3 = 7	7 − 1 = 6

Week 2		Practice Page 3
10 − 1 = 9	8 − 2 = 6	8 − 4 = 4
7 − 4 = 3	9 − 3 = 6	3 − 1 = 2
4 − 2 = 2	6 − 3 = 3	10 − 4 = 6
6 − 1 = 5	9 − 4 = 5	8 − 3 = 5
7 − 3 = 4	7 − 2 = 5	5 − 2 = 3
10 − 3 = 7	7 − 1 = 6	9 − 2 = 7

Week 2		Practice Page 4
10 − 2 = 8	6 − 3 = 3	10 − 4 = 6
5 − 1 = 4	9 − 4 = 5	8 − 3 = 5
9 − 1 = 8	6 − 2 = 4	8 − 4 = 4
7 − 4 = 3	9 − 3 = 6	7 − 2 = 5
7 − 3 = 4	2 − 1 = 1	4 − 1 = 3
10 − 3 = 7	8 − 1 = 7	3 − 2 = 1